ROUTE OPTIMIZATION OF MULTIPLE-AGENT TRAVELLING SALESMAN PROBLEM

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ABSTRACT

Style: Route optimization is quotidian engineering problem. Problem of finding the optimal and sub-optimal routes is one of the most studied optimization problem. In this paper, author firstly presented a short literature overview of the history of routing problems and namely travelling salesman problem (TSP). In the second chapter author presented the usual mathematical formulations for single and multiple agent travelling salesman problem (TSP and mTSP). In chapter three, author used a case study TSP given in the literature which involves one agent and fifteen nodes, modelled the problem in a standard software package (MS Excel's Visual Basic VBA), solved it by the evolutionary solver in the same software package, confirmed the result and withal verified the model. In the following step, author added one more agent as a hypothetical case of mTSP and solved the problem. In the final chapter author discussed the results and gave conclusions which can be used for further development of the study. The solution was gained in reasonably short computational time and considered as optimal.

Key words: *route optimization, travelling salesman problem, multiple-agent, evolutionary algorithm.*

1 INTRODUCTION

Finding the shortest travel route is perhaps the clearest and most common natural objective. Nowadays it is a matter of few seconds to find all feasible routes by the usage of internet maps and it seems as a trivial problem. However, even today it can be mathematically challenging problem. It is believed that it was originally mathematically introduced by Euler in mid-18th century in his works for solving travelling paths from and to Prussian city of Königsberg (present-day Kaliningrad, Russia) [1], and for solving knight's tour [2]. Almost simultaneously, there have been developed similar studies and mathematical analysis of graph theories, which are systematically elaborated in [3].

Perhaps the most known routing problem is the travelling salesman problem (TSP). TSP is defined as a combinatorial optimization problem of finding the shortest route for an agent (i.e. travelling salesman) on his trip starting from his home to visit each city exactly once before returning home [4]. Even though it is considered that the problem was introduced as the TSP in 1930's, it was actually structured and published in German travelling salesman handbook in 1832 [5]. It is accepted that TSP was presented to worldwide scientific stage by Merrill Flood in his paper [6] in 1956. However, it took twenty years until the solution was presented in the paper [7], which had drawn attention of the scientific circles just as much as its mathematical formulations.

TSP can vary from a small to large complex problem. Its complexness is supported by the fact that even today, there isn't a non-deterministic polynomial-time algorithm for solving TSP, and hence it is considered as a NP-complete problem. Therefore, it is a relevant mathematical optimization problem today. As it was previously stated, routing problem is a natural problem and as such can be found in animal world as well. While humans tend to have fuzzy and sometimes even illogical constraints, in animal world routing problems are logical and clear. Therefore, nowadays there are numerous bio-inspired heuristic and meta-heuristic algorithms for solving TSP's. Among others, there are ant-colony optimization algorithm that was presented in [8], bee colony optimization algorithm [9], cuckoo search algorithm [10], mosquito host-seeking algorithm [11], bat algorithm [12], discrete firefly algorithm [13], etc.

TSP, by its original structure, is a problem of a single agent which is obvious even by its name (i.e. travelling salesman is a singular). However, in literature multiple-agent TSP (mTSP) can be found, and it is considered as an appropriate TSP modification for real-time application. Formulations and solution procedures of mTSP is presented in a comprehensive overview given in [14].

For construction industry, TSP among other routing problems is very interesting. Problems such as planning and sequencing construction resources, trajectories on the construction sites, raw and prefabricated material and elements delivery are everyday tasks for construction managements. There are numerous papers and studies presenting routing problems and specifically TSP's in construction. For the sake of the length of the paper author here presents some most recent papers published in respected journals. In paper [15] authors presented an optimization model for tower crane operation efficiency improvement by the usage of TSP formulation. The results of the model's application have shown a significant savings in total travel time of a tower crane. Authors [16] presented a route reliability model for asphalt delivery in urban areas, while similar model was presented in [17] for solving ready-mix concrete delivery problem. For construction monitoring, authors [18] developed a TSP based inspection path for unmanned aerial vehicle (UAV). In paper [19] authors presented a TSP leaned model for sewer inspection. For earthwork allocation problem authors [20] developed a model oriented on the least-cost route. For the road network daily maintenance routing problem with uncertain service time, authors [21] proposed a robust optimization model and solved the problem by the branch-and-cut method. In paper [22] author structured a TSP for construction site supervision consisted of 15 sites for one supervisor to visit. Author used a General Algebraic Modelling System (GAMS) for modelling the problem, while for its solving authors used mixed integer linear programing in the computer package CPLEX.

In this paper author presents a simpler approach for modelling and solving the example of TSP presented in [22], by using the Microsoft Excel. As a further step, author added one more agent in the example by which TSP becomes mTSP, whilst m equals 2. For modelling both problems, author used Microsoft Excel VBA, while for solving both version of problem evolutionary algorithm was used. The result of the original TSP was confirmed and in the case of mTSP the results was gained in reasonably short computational time and it is considered as an optimal result.

2 MATHEMATICAL FORMULATIONS OF THE TSP AND MTSP

Single and multiple-agent TSP can be symmetric (TSP) or asymmetric (aTSP) in their matrix form. Graphically TSP is described as complete undirected graph $G = (V, E)$, where V is a vertex set $V=\{1, \dots, n\}$, and E denotes an edge set $E=\{(i, j): i, j \in V, i < j\}$ [23]. Constant values of distances from one edge to another are given as a matrix $D=(d_{ij})$ and it has to satisfy the triangle inequality $d_{ij} \leq d_{ik} + d_{kj}$ for all i, j and k, whilst d_{ij} is the shortest distance from i to j on graph G. This is the case where d_{ij} is consider as Euclidean distance.

The common mathematical formulation of the TSP, given as integer programming structure, is defined by the objective function of minimization of the total travel length (equation 1):

$$\min Z = \sum_{i<j} \mathrm{d}_{ij} \cdot x_{ij} \tag{1}$$

Subject to constraints (equation 2, 3 and 4):

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2 \tag{2}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \quad (S \subset V, 3 \leq |S| \leq n-3) \tag{3}$$

$$x_{ij} = 0 \text{ or } 1 \quad (i,j) \in E \tag{4}$$

In the case of the mTSP mathematical formulation, there are several formulations in literature. Here author will present a formulation for the symmetric mTSP, given in [24]. Minimization of the objective function (eq. 5):

$$\min Z = \sum_{i<j} \mathrm{d}_{ij} \cdot x_{ij} + f_m \tag{5}$$

Subject to constraints (eq. 6 to 11):

$$\sum_{j=2}^{n} x_{1j} = 2m \tag{6}$$

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2\,, \quad k = 2, \dots, n \tag{7}$$

$$\sum_{i<j;\, i,j \in S} x_{ij} \leq |S-1|,\ 3 \leq |S| \leq n-2,\ S \subseteq V \setminus \{1\} \tag{8}$$

$$x_{ij} \in \{0,1\},\ 1 < i < j \tag{9}$$

$$x_{1j} \in \{0,1,2\},\ j = 2, \dots, n \tag{10}$$

$$m \in \mathbb{N}_{>1} \tag{11}$$

Mathematical formulation for asymmetric problems, as well as alternative mathematical formulation to those here presented, can be found in [23]. In this paper, author used the above presented mathematical formulations for modelling the case study problem and its modification.

3 OPTIMIZATION MODEL FOR SOLVING SMALL AND MEDIUM TSP AND MTSP

3.1 Optimization model for modelling and solving TSP in Excel

In this study, author used Microsoft Excel, a standard engineering tool, for modelling and solving the addressed optimization problem. Excel add-inn solver is a useful tool, however it is limited by the size of the problem (i.e. 200 decision variables and constraints) by which it is applicable for small and medium size problems [25]. In order to avoid this limitation, author used basic functions in Excel. Instead of decision matrix as variables, suggested by the mathematical formulations given earlier in the text, author used Excel's *index* function.

Variables are numerated nodes (edges) and by the *index* function belonging input parameters (i.e. distances d_{ij}) are returned (Tab. 1). By this manoeuvre, author avoided accumulation of binary variables, and the model is limited by the number of nodes *n*. Constraint variables (i.e. nodes) were defined as *all different* and integer (*i*, *j*, *k* = 2, ..., *n*). Since TSP usually has determined starting node, author assumed node 1 as a starting and destination node.

nodes	1	*i*	*j*	*k*	1
distances	d_{1i}	d_{ij}	d_{jk}	d_{k1}	

Tab. 1 Optimization model for solving TSP

Optimization model for solving mTSP is modified and shown in Tab. 2:

1	nodes	1	*i*	*j*	*k*	1
	distances	d_{1i}	d_{ij}	d_{jk}	d_{k1}	
m	nodes	1	*i*	*j*	*k*	1
	distances	d_{1i}	d_{ij}	d_{jk}	d_{k1}	

Tab. 2 Optimization model for solving mTSP

The objective functions were defined as they are given in eq. 1 for TSP, and in eq. 5 in case of the mTSP. Evolutionary algorithm was used for solving problems, with the mutation rate of 0,075; population size of 100 and maximum time without solution improvement 300 seconds.

3.2 Application of the model for solving single-agent TSP

For the application of the model, author used a case study problem presented in the paper [22] consisted of 15 nodes (i.e. construction sites in Slovenia) where construction supervisor has to check each node, starting and ending at node 1.

d_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1000	25,2	55,4	1000	1000	13,1	56,8	1000	32,4	25,6	17,9	1000	9,4	11,4	13,7
2	25,2	1000	41,5	1000	1000	17,6	1000	1000	1000	1000	1000	1000	1000	1000	1000
3	55,4	41,5	1000	14,8	6,8	42,7	22,0	1000	47,2	1000	1000	1000	1000	1000	1000
4	1000	1000	14,8	1000	14,9	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
5	1000	1000	6,8	14,9	1000	1000	24,8	1000	1000	1000	1000	1000	1000	1000	1000
6	13,1	17,6	42,7	1000	1000	1000	44,5	1000	25,7	1000	1000	1000	1000	18,6	1000
7	56,8	1000	22,0	1000	24,8	44,5	1000	17,4	33,9	1000	1000	1000	1000	1000	1000
8	1000	1000	1000	1000	1000	1000	17,4	1000	23,7	1000	1000	1000	1000	1000	1000
9	32,4	1000	47,2	1000	1000	25,7	33,9	23,7	1000	25,1	1000	21,1	1000	24,9	1000
10	25,6	1000	1000	1000	1000	1000	1000	1000	25,1	1000	12,6	12,3	19,4	1000	1000
11	17,9	1000	1000	1000	1000	1000	1000	1000	1000	12,6	1000	5,9	9,3	11,9	1000
12	1000	1000	1000	1000	1000	1000	1000	1000	21,1	12,3	5,9	1000	5,1	5,1	1000
13	9,4	1000	1000	1000	1000	1000	1000	1000	1000	19,4	9,3	5,1	1000	2,7	16,8
14	11,4	1000	1000	1000	1000	18,6	1000	1000	24,9	1000	11,9	5,1	2,7	1000	1000
15	13,7	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	16,8	1000	1000

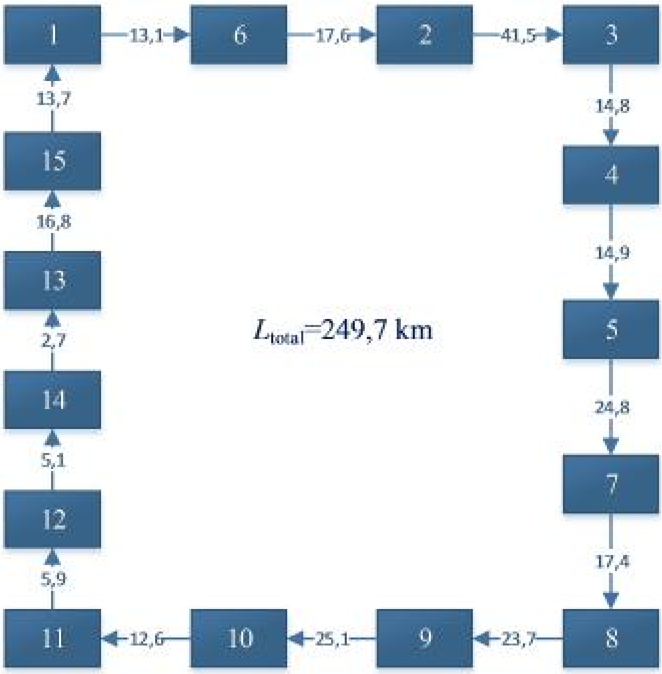
Tab. 3 Distance matrix between nodes [km] [22]

Distances presented in the original study form a symmetric matrix (see Tab.3), which makes this problem suitable for the application of the mathematical formulation given by equation 1 to 4 for its modelling.

It is important to underline that in the original case study problem [22] author used the infinity to define infeasible routes, while in this study author used relatively large distances (i.e. 1000 km) since the objective function is to minimize the total length of the route. It is necessary to keep in mind that when using VBA, empty cells might be defined as zero values, which can compromise the optimization process.

Case study problem, taken here as the model verification example, is considered as a small-scale problem. Usually in such cases, it is not necessary to start the optimization process with an initial feasible solution. However, because author used an evolutionary algorithm for solving the problem and the computational time can be significantly decreased by giving the initial solution, author set the initial solution with the nodes in numerical order.

The starting and finishing node was given as it was in the original study (i.e. node 1); there are total 14 variables (Fig. 1).

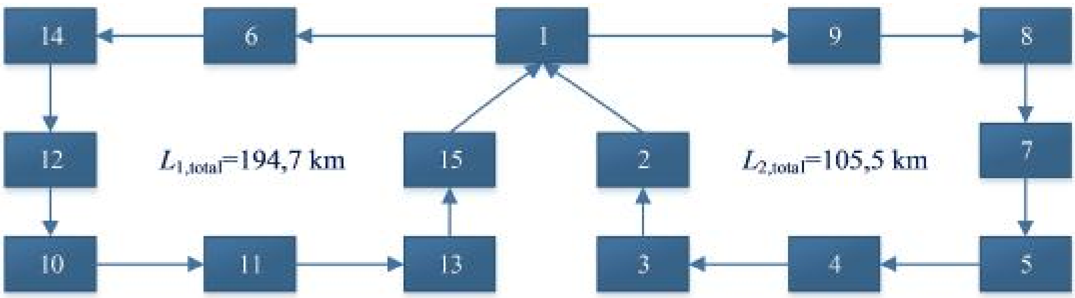


Fig, 1 Optimal route for the single-agent case study TSP

The result was gained in reasonably short time and it is equal to the one presented in the original paper. Total travelled distance is 249,7 km with the path same as it was gained in the original paper.

3.3 Application of the model for solving two-agent TSP

In extension of the original problem, author added one more agent. Hence, the problem becomes the mTSP (i.e. m=2). The input parameters were the same as in previous and original case. Both agents were determined to go from and to return to node 1, therefore from total 14 variables (nodes) each agent had 7 nodes assigned (Fig 2).



Fig, 2 Optimal routes for the two-agent case study TSP

As in the previous case, the result was gained rather quickly. Total travel distance for one of the agents has to go 105,5 km while for the 194,7 km. Their total travel distance is 300,2 km, which is longer than in the first case, as expected. However, the premise is that two agents can travel simultaneously and comparing the result of the case with only one agent and the result of the agent which has a longer route there is a significant saving in travel distance.

4 DISCUSSION AND CONCLUSIONS

Routing optimization problems are natural and applied quotidian engineering problems. Among numerous algorithms and tools for solving these types of problems, the best ones are considered those that tend to be as simple as possible. Their simplicity is mostly characterized in the modelling language and tools. Therefore, in this paper author used a common engineering tool, i.e. Microsoft Excel, for modelling and solving a literature case study TSP by the usage of the evolutionary algorithm. The result gained in this study was the same as it was in the original study by which the model was verified. In extension of the problem, author added one more agent to the problem and gained a result, which is considered as optimal. Solutions were gained reasonably quickly and most importantly the effort for modelling in both variants of the problem is not time consuming or requires advanced knowledge of programming language.

In further development of the model, it should be tested on larger, asymmetric and problems with more agents. As well, by adding more agents the TSP becomes an mTSP, which is considered as a more realistic variant of the problem. Appointing the optimal number of agents in regard to the time and costs of engaged agents is typical queuing theory (QT) problem. In combination of these approcahes a multiple objective optimization problem can be structured consisted of assignment problem and TSP, while the utilization of each agents should be maximized.

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